

On the communication complexity of certain hypergraph vertex-colouring games

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We define vertex-colouring games for hypergraphs along lines similar to Kochen-Specker games and investigate their communication complexity. Given any labelled n -vertex r -uniform hypergraph G , Alice is given an arbitrary hyperedge h and Bob is given an arbitrary vertex v in the same hyperedge h . Alice must colour each vertex in hyperedge h black or white, and Bob must colour the vertex v black or white. Alice also has to obey the rule that exactly one of the vertices in h is coloured white. The game is won if the vertex v gets the same colour from both Alice and Bob. We develop the notion of "colouring strategies" to establish lower bounds on the communication complexity of winning such a game. We derive a lower bound of $\log(n-r+1)$ for r -uniform complete hypergraphs, for $r < n < 2r$. We design optimal protocols matching this bound. For the case $r=2$ (ordinary complete graphs) the optimal communication complexity is as low as $\log \log n$.